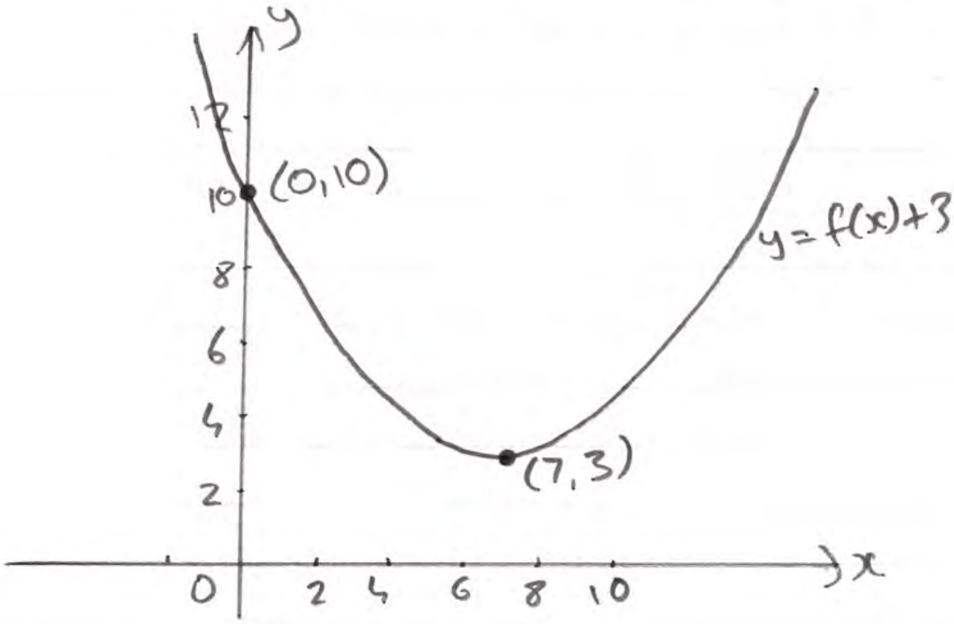
# C1 June 2008 (MA)

Q1) $\int (2+5x^2)\,dx = \boxed{2x + \frac{5x^3}{3} + c}$

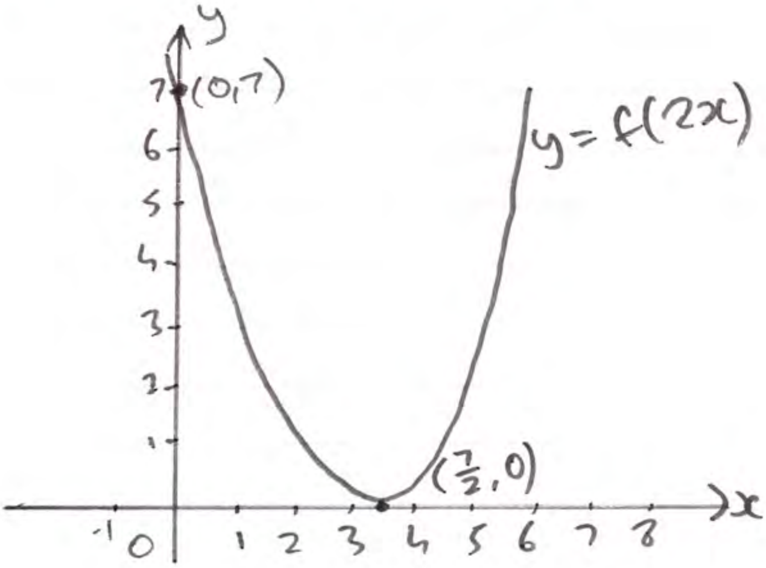
Q2) $x^3 - 9x = x(x^2-9)$

$= \boxed{x(x-3)(x+3)}$

Q3a) $y=f(x)+3$ - transformation vertically of '+3'




b) $y=f(2x)$ - halve the $x$-coordinates:



Q4) $f(x) = 3x + x^3, \; x > 0$

a) $\boxed{f'(x) = 3 + 3x^2}$

b) Given $f'(x) = 15$,

then $15 = 3 + 3x^2$

$12 = 3x^2$

$4 = x^2$

$x = \pm 2$

But $x > 0$, so $\boxed{x = 2}$

Q5) $x_1 = 1$

$x_{n+1} = ax_n - 3, \; n \geq 1$

a) $x_2 = a(x_1) - 3$

$\boxed{x_2 = a - 3}$

b) $x_3 = ax_2 - 3$

$x_3 = a(a-3) - 3$

$\boxed{x_3 = a^2 - 3a - 3}$

c) Given $x_3 = 7$,

then $7 = a^2 - 3a - 3$

$$a^2 - 3a - 10 = 0$$

$$(a-5)(a+2) = 0$$

$$\boxed{\text{Either } a=5 \quad \text{or} \quad a=-2}$$

Q6a) $y = \frac{3}{x}$ : When $x \to \infty$, $y \to 0$
When $x \to -\infty$, $y \to -0$

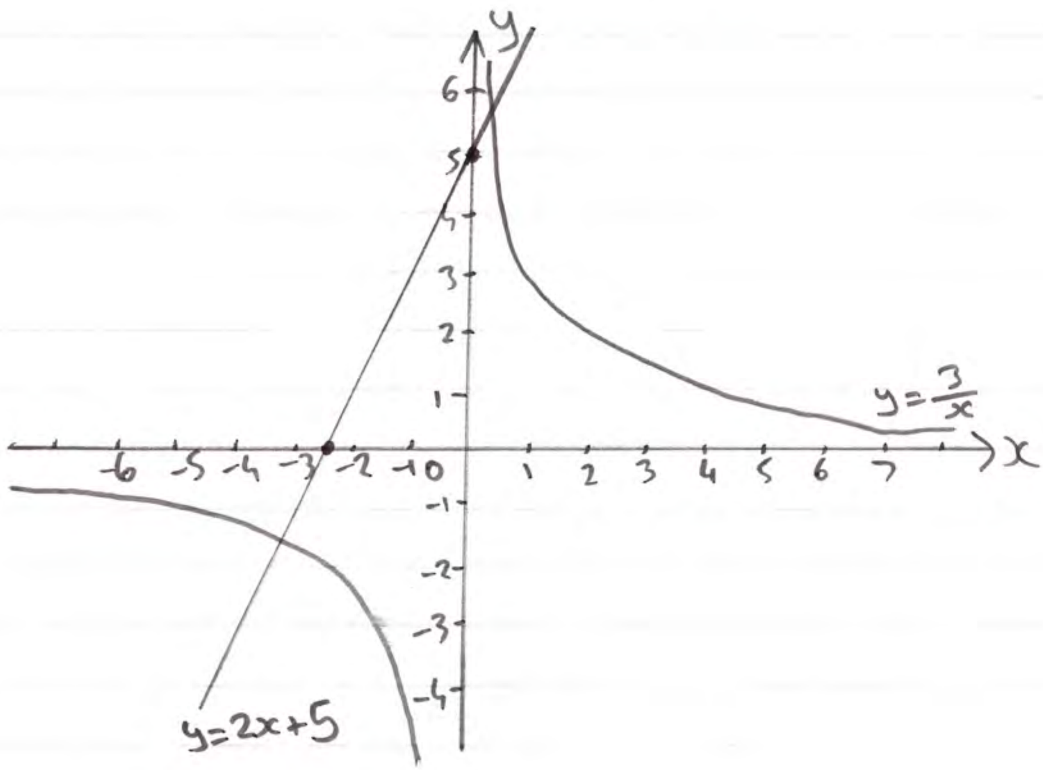
When $x \to 0$, $y \to \infty$
When $x \to -0$, $y \to -\infty$

$y = 2x+5$ : When $\underline{x=0, \ y=5}$

When $y=0$, $2x+5=0$

$$2x = -5$$

$$\underline{x = -\frac{5}{2}}$$



b) To find points of intersection, solve simultaneous equations:

$$y = 2x + 5 \quad ①$$
$$y = \frac{3}{x} \quad ②$$

Substitute ① into ②:

$$2x + 5 = \frac{3}{x}$$

$$2x + 5 - \frac{3}{x} = 0$$

$$\frac{2x^2 + 5x - 3}{x} = 0$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

Either $x = \frac{1}{2}$ or $x = -3$

Substitute into ② for $y$:

When $x = \frac{1}{2}$, $y = \frac{3}{\frac{1}{2}}$ $\underline{= 6}$

When $x = -3$, $y = \frac{3}{-3}$ $\underline{= -1}$

The points of intersection are:

$$\boxed{\left(\frac{1}{2}, 6\right) \text{ and } (-3, -1)}$$

Q7) First term, $a = 5km$
Common difference, $d = 2km$

a) $U_n = a + (n-1)d$

$U_4 = 5 + (4-1)2$

$U_4 = 5 + 6$

$\boxed{U_4 = 11\,km}$

b) $U_n = a + (n-1)d$

$U_n = 5 + (n-1)d$

$U_n = 5 + (n-1)2$

$U_n = 5 + 2n - 2$

$U_n = 3 + 2n$

$\boxed{U_n = 2n + 3}$

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = \frac{n}{2}(2(5) + (n-1)2)$

$S_n = \frac{n}{2}(10 + 2n - 2)$

$$S_n = \frac{n}{2}(2n+8)$$

$$S_n = \frac{2n^2}{2} + \frac{8n}{2}$$

$$S_n = \frac{2n^2+8n}{2}$$

$$S_n = n^2+4n$$

$$\therefore \boxed{S_n = n(n+4)\text{ km}}$$

d) $43 = 2n+3$

$43-3=2n$

$2n=40$

$$\boxed{n=20}$$

e) $S_{20} = 20(20+4)$

$S_{20} = 20 \times 24$

$$\boxed{S_{20} = 480\text{ km}}$$

Q8) $2qx^2+qx-1=0$ has no real roots

a) If a quadratic has no real roots, then the discriminant is less than 0.

$\therefore\ b^2-4ac<0$

$q^2-(4)(2q)(-1)<0$

$q^2-8q(-1)<0$

$\therefore\ \boxed{q^2+8q<0}$

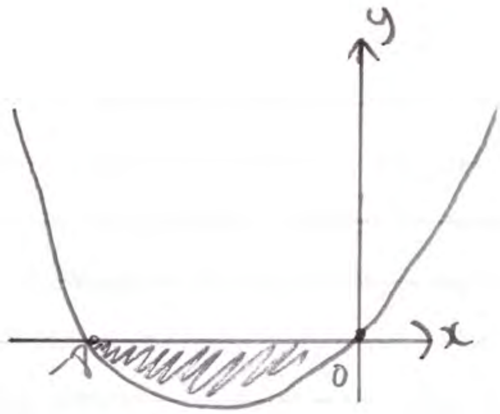
b) For $q^2+8q=0$,

$q(q+8)=0$

Either $q=0$ or $q=-8$

Set of possible values of $q$:

$\boxed{-8<q<0}$




Choosing the value 'under' the $x$-axis, since $q^2+8q<0$

Q9a) $y=kx^3-x^2+x-5$

$\boxed{\dfrac{dy}{dx}=3kx^2-2x+1}$

b) When $x = -\frac{1}{2}$, $\frac{dy}{dx} = 3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1$

$$= 3k\left(\frac{1}{4}\right) + 1 + 1$$

$$= \frac{3k}{4} + 2$$

$\therefore$ the gradient of the tangent at $A = \frac{3k}{4} + 2$

For the line $2y - 7x + 1 = 0$,

$$2y = 7x - 1$$

$$y = \frac{7}{2}x - \frac{1}{2}$$

The gradient is $\frac{7}{2}$

Since the tangent at A is parallel to $2y - 7x + 1 = 0$, then both will have gradients of $\frac{7}{2}$.

$$\therefore \frac{3k}{4} + 2 = \frac{7}{2}$$

$$\frac{3k}{4} = \frac{3}{2}$$

$$3k = 6$$

$$\boxed{k = 2}$$

c) When $x = \frac{-1}{2}$, $y = 2\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 5$

$$y = 2\left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5$$

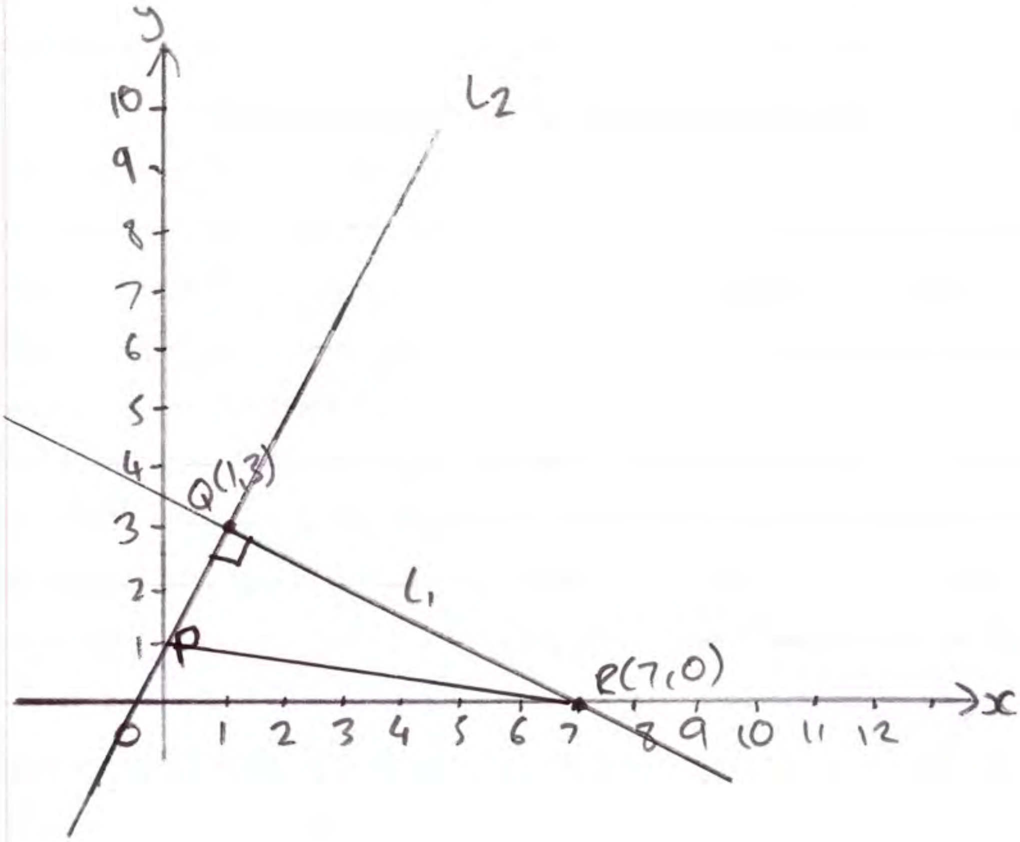
$$y = -\frac{2}{8} - \frac{1}{4} - \frac{1}{2} - 5$$

$$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5$$

$$\underline{y = -6}$$

$\therefore$ the coordinates of A are $\boxed{\left(-\frac{1}{2}, -6\right)}$

Q10)



a) Length QR $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-1)^2+(0-3)^2}$

$= \sqrt{6^2+(-3)^2}$

$= \sqrt{45} = \sqrt{9\times 5} = \sqrt{9}\sqrt{5}$

$= \underline{3\sqrt{5} \text{ units}}$

$\therefore \boxed{a=3}$

b) Gradient QR $= \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{0-3}{7-1}$

$= \dfrac{-3}{6}$

$= \underline{-\dfrac{1}{2}}$

$\therefore$ the Gradient of the perpendicular line, $l_2$ is $\underline{2}$

Equation of $l_2$: $y-y_1 = m(x-x_1)$

Using Q(1, 3) and m=2 $\rightarrow$ $y-3 = 2(x-\mathbf{z})$

$y-3 = 2x-2$

$\boxed{y = 2x+1}$

c) when $l_2$ crosses the y-axis, $x = 0$

$$\therefore \; y = 2(0) + 1 \implies y = 1$$

$$\therefore \boxed{P \text{ is at } (0,1)}$$

d) Length $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(1-0)^2 + (3-1)^2}$$

$$= \sqrt{1^2 + 2^2}$$

$$\boxed{= \sqrt{5} \text{ units}}$$

Area $\Delta PQR = \frac{1}{2} bh = \frac{1}{2}(QR)(PQ)$

$$= \frac{1}{2}(3\sqrt{5})(\sqrt{5})$$

$$= \frac{1}{2}(15)$$

$$= \boxed{\frac{15}{2} \text{ units}^2}$$

Q11a) $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}, x \neq 0$

$$\frac{dy}{dx} = \frac{(x^2+3)(x^2+3)}{x^2}$$

$$\frac{dy}{dx} = \frac{x^4+6x^2+9}{x^2}$$

$$\frac{dy}{dx} = \frac{x^4}{x^2} + \frac{6x^2}{x^2} + \frac{9}{x^2}$$

$$\boxed{\frac{dy}{dx} = x^2 + 6 + 9x^{-2}}$$

b) $y = \int (x^2+6+9x^{-2})\,dx$

$$y = \frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1} + c$$

$$y = \frac{x^3}{3} + 6x - \frac{9}{x} + c$$

Since the curve passes through (3,20), substitute in $x=3$ and $y=20$:

$$20 = \frac{3^3}{3} + 6(3) - \frac{9}{3} + c$$

$$20 = \frac{27}{3} + 18 - 3 + c$$

$$20 = 9 + 18 - 3 + c$$

$\therefore c = -4 \implies \boxed{y = \frac{x^3}{3} + 6x - \frac{9}{x} - 4}$